

List of Books/Chapters Edited

Sr. No.	Particulars
1	Dr. Ravish R. Singh, Basic Electrical Engineering, S Chand Publishing, 2022.
2	Dr. Ravish R. Singh, Mathematics I, S Chand Publishing, 2022.
3	Dr. Mukul Bhatt, Mathematics I, S Chand Publishing, 2022.
4	Dr. Ravish R. Singh, Mathematics II, S Chand Publishing, 2022.
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9	Dr. Ravish R. Singh, Probability and Statistics, S Chand Publishing, 2022.
10	Dr. Mukul Bhatt, Probability and Statistics, S Chand Publishing, 2022.
11	Dr. Ravish R Singh, Engineering Mathematics Vol. I, McGraw Hill Education, India, 2020.
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44	Dr. Ravish R. Singh, Implementation of ISO 9001:2015 International Standard in Teaching Learning Process of TRCL, Infinity Publishing, India, 2018.
45	Dr. Mukul Bhatt, Implementation of ISO 9001:2015 International Standard in Teaching Learning Process of TRCL, Infinity Publishing, India, 2018.

Basic Electrical Engineering

Written lucidly, **Basic Electrical Engineering** is designed specifically for the first-year engineering students at the University of Mumbai. In that, the positive aspect is a thoughtful blend of theory and problems. This not only helps the students understand the concepts explained but also increases their practice quotient.

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- Follows Bloom's taxonomy (Specific learning outcomes can be derived from the taxonomy, though it is oft used to assess learning on a variety of cognitive levels.)
- Apt coverage with strict adherence to the MU syllabus of Basic Electrical Engineering
- Completion of each section is accompanied with multi-format exercises to test gleaning of individual subject matter
- A rich exam-oriented pedagogy includes:
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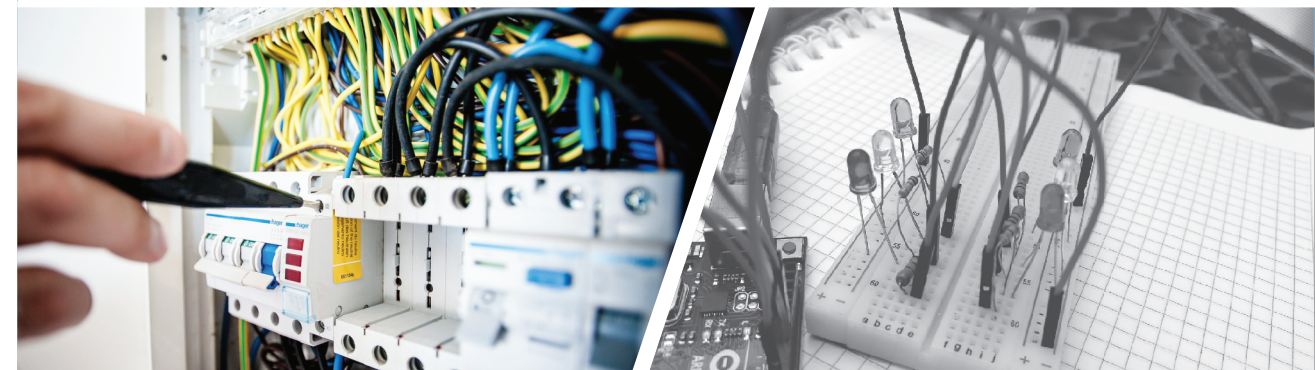
Basic Electrical Engineering

RAVISH R SINGH



Basic Electrical Engineering

University of Mumbai



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RAVISH R SINGH

Basic Electrical Engineering

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Engineering among others for all-India curricula as well as regional curricula of universities such as Gujarat Technological University and Mumbai University among many others. Dr Singh is a member of Institute of Electrical and Electronics Engineers (IEEE), Indian Society for Technical Education (ISTE) and Institution of Electronics and Telecommunication Engineers (IETE) and has to his credit several published research papers in national and international journals. His fields of interest include Circuits, Signals and Systems and Engineering Mathematics.

Basic Electrical Engineering

Ravish R Singh

Director

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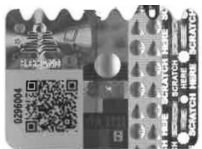
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First Edition 2023

ISBN: 978-93-5501-536-5

Product Code: H3BEE52BEEL10ENAA230

PRINTED IN INDIA

By Vikas Publishing House Private Limited, Plot 20/4, Site-IV, Industrial Area Sahibabad, Ghaziabad – 201 010 and Published by S Chand And Company Limited, A-27, 2nd Floor, Mohan Co-operative Industrial Estate, New Delhi – 110 044.

Dedicated
to
My son, *Aman*
and
daughter, *Aditri*

Preface

Basic Electrical Engineering, as a subject, encompasses within itself the core understanding of major concepts of the subject, including but not limited to, DC and AC Circuits, Transformers, Electrical Machines and DC Machines apart from Kirchhoff's laws, Norton's theorem and principle of operation of single-phase induction motors among many others.

Written lucidly, Basic Electrical Engineering is designed specifically for the first-year engineering students at the University of Mumbai. In that, the positive aspect is a thoughtful blend of theory and problems. This not only helps the students understand the concepts explained but also increases their practice quotient.

Salient Features:

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 - Close to 400 exercise question

ACKNOWLEDGEMENTS

My acknowledgements would be incomplete without a mention of the contribution of my family members. I am indebted to my mother and father for their lifelong inspiration. A heartfelt thanks is due to my wife, Nitu; son, Aman; and daughter, Aditri, for always motivating and supporting me in the whole project.

Any suggestions for improving the book will be gratefully acknowledged.

Ravish R Singh

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Basic Circuit Concepts – Prerequisite

1

LEARNING OBJECTIVES

After studying this chapter, you should be able to:

- LO 1.1** Explain the concepts of voltage, current, power and energy
- LO 1.2** Explain the concepts of resistance, inductance, and capacitance
- LO 1.3** Analyse series and parallel connections of resistances
- LO 1.4** Discuss sources and their types
- LO 1.5** Define networks and circuits, linear and non-linear elements, active and passive elements, unilateral and bilateral elements, lumped and distributed elements, active and passive networks, time-invariant and time-variant networks
- LO 1.6** Define magnetic circuits, magnetomotive force, ampere-turns, magnetic field strength, reluctance and permeance
- LO 1.7** Explain the concepts of series and parallel magnetic circuits
- LO 1.8** Explain the concepts of magnetic leakage and fringing
- LO 1.9** Describe BH curves
- LO 1.10** Explain time-domain analysis of first order *RL* and *RC* circuits

1.1 || VOLTAGE

We know that like charges repel each other whereas unlike charges attract each other. To overcome this force of attraction, a certain amount of work or energy is required. When the charges are separated, it is said that a potential difference exists and the work or energy per unit charge utilized in this process is known as voltage or potential difference.

$$V = \frac{\text{work done}}{\text{charge}} = \frac{W}{Q}$$

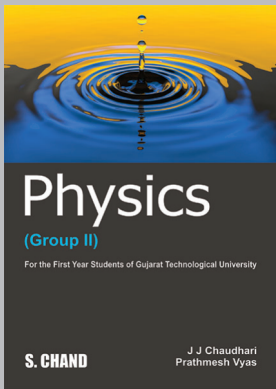
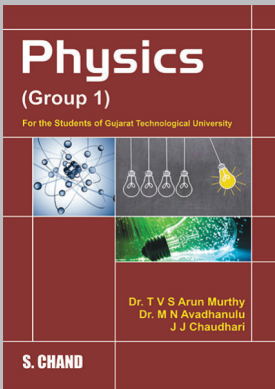
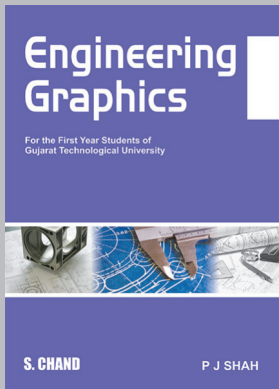
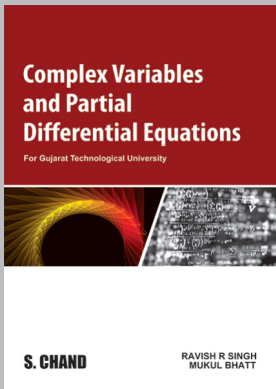
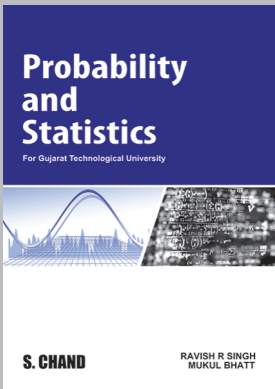
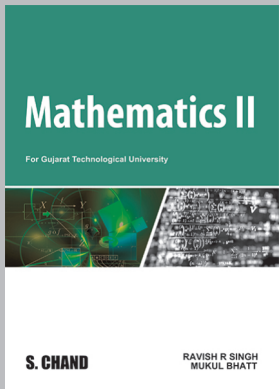
Voltage is measured in volts (V).

LO 1.1

Explain the concepts of voltage, current, power and energy

Mathematics I

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Engineering among others for all-India curricula as well as regional curricula of universities such as Gujarat Technological University and Mumbai University among many others. Dr Singh is a member of Institute of Electrical and Electronics Engineers (IEEE), Indian Society for Technical Education (ISTE) and Institution of Electronics and Telecommunication Engineers (IETE) and has to his credit several published research papers in national and international journals. His fields of interest include Circuits, Signals and Systems and Engineering Mathematics.



Mukul Bhatt, M. Sc. (Mathematics) (H N B Garhwal University), Ph.D. (PAHER University, Udaipur) is Assistant Professor, Department of Mathematics and Statistics, Thakur Ramnarayan College of Arts and Commerce, Mumbai. Dr. Bhatt has close to three decades of teaching experience at various levels in engineering colleges and she has published several

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Mathematics I

For Gujarat Technological University

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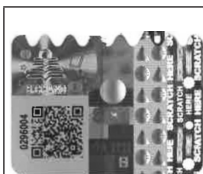
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First Edition 2023

ISBN: 978-93-5501-528-0

Product Code: H6MGT61MATH10ENAA23O

PRINTED IN INDIA

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and Published by S Chand And Company Limited, A-27, 2nd Floor, Mohan Co-operative Industrial Estate,
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**Dedicated
to**

Aman and Aditri

Ravish R Singh

Soumya and Siddharth

Mukul Bhatt

Preface

Some would argue that Mathematics is the cornerstone of engineering. They will not be wrong. Mathematics is the science and study of quality, structure, space and change. Its students seek patterns, formulate new conjectures and establish truth by rigorous deduction from appropriately chosen axioms and definitions. Development of analytical skills is an easy by-product of studying mathematics which in real-life situations is a boon to have.

“**Mathematics I**” has been designed specifically for the first year Gujarat Technological University (GTU) syllabus and students of all programmes of engineering since first semester mathematics is common to all branches.

Ten dedicated chapters and five appendices are set to sequentially cover each module of the syllabus and are compounded by the ‘tutorial technique’, i.e., theory followed by example(s) so that the learner develops an increased sense of conscious intellection.

This exceptional mix of theory and application caters to all types of requirements, be it the student or the teacher. Not only is the syllabus rigorously followed, but each topic has also been treated with the end-examination in sight. Concepts are well-aided with solved examples (of different complexities) so that every learner understands the topic at hand. Care has been taken for appropriate incorporation of Solutions of GTU examination questions within the text.

Salient Features:

- Apt coverage with strict adherence to the latest GTU syllabus of Mathematics-1.
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 - More than 550 exercise questions

Ravish R Singh
Mukul Bhatt

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CHAPTER 1

Indeterminate Forms

Chapter Outline

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- 1.3 Type 1 : $\frac{0}{0}$ Form
- 1.4 Type 2 : $\frac{\infty}{\infty}$ Form
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1.1 INTRODUCTION

We have studied certain rules to evaluate the limits. But some limits cannot be evaluated by using these rules. These limits are known as indeterminate forms. There are seven types of indeterminate forms:

- (i) $\frac{0}{0}$
- (ii) $\frac{\infty}{\infty}$
- (iii) $0 \times \infty$
- (iv) $\infty - \infty$
- (v) 1^∞
- (vi) 0^0
- (vii) ∞^0

These limits can be evaluated by using L'Hospital's rule.

1.2 L'HOSPITAL'S RULE

Statement If $f(x)$ and $g(x)$ are two functions of x which can be expanded by Taylor's series in the neighbourhood of $x = a$ and if $\lim_{x \rightarrow a} f(x) = f(a) = 0$, $\lim_{x \rightarrow a} g(x) = g(a) = 0$, then

Mathematics II

For Gujarat Technological University



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**RAVISH R SINGH
MUKUL BHATT**

Mathematics II

For Gujarat Technological University

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Engineering among others for all-India curricula as well as regional curricula of universities such as Gujarat Technological University and Mumbai University among many others. Dr Singh is a member of Institute of Electrical and Electronics Engineers (IEEE), Indian Society for Technical Education (ISTE) and Institution of Electronics and Telecommunication Engineers (IETE) and has to his credit several published research papers in national and international journals. His fields of interest include Circuits, Signals and Systems and Engineering Mathematics.



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Mathematics II

For Gujarat Technological University

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Registered Office: A-27, 2nd Floor, Mohan Co-operative Industrial Estate,
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First Edition 2023

ISBN: 978-93-5501-562-4

Product Code: H6MGT61MATH10ENAA230

PRINTED IN INDIA

By Vikas Publishing House Private Limited, Plot 20/4, Site-IV, Industrial Area Sahibabad, Ghaziabad – 201 010
and Published by S Chand And Company Limited, A-27, 2nd Floor, Mohan Co-operative Industrial Estate,
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**Dedicated
to**

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Ravish R Singh

Soumya and Siddharth

Mukul Bhatt

Preface

Some would argue that Mathematics is the cornerstone of engineering. They will not be wrong. Mathematics is the science and study of quality, structure, space and change. Its students seek patterns, formulate new conjectures and establish truth by rigorous deduction from appropriately chosen axioms and definitions. Development of analytical skills is an easy by-product of studying mathematics which in real-life situations is a boon to have.

“**Mathematics II**” has been designed specifically as per the Gujarat Technological University (GTU) syllabus and students of all programmes of engineering.

Six dedicated chapters are set to sequentially cover each module of the syllabus and are compounded by the ‘tutorial technique’, i.e., theory followed by example(s) so that the learner develops an increased sense of conscious intellection.

This exceptional mix of theory and application caters to all types of requirements, be it the student or the teacher. Not only is the syllabus rigorously followed, but each topic has also been treated with the end-examination in sight. Concepts are well-aided with solved examples (of different complexities) so that every learner understands the topic at hand. Care has been taken for appropriate incorporation of Solutions of GTU examination questions within the text.

Salient Features:

- Apt coverage with strict adherence to the latest GTU syllabus of Mathematics II.
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 - Close to 700 in-text solved examples and Figures
 - Close to 550 exercise questions

Ravish R Singh
Mukul Bhatt

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CHAPTER 1

Vector Calculus

Chapter Outline

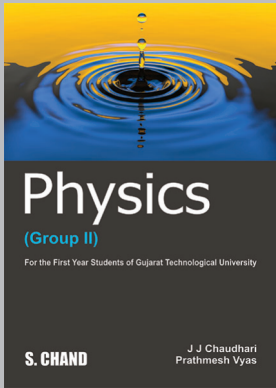
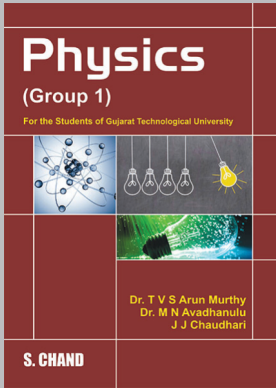
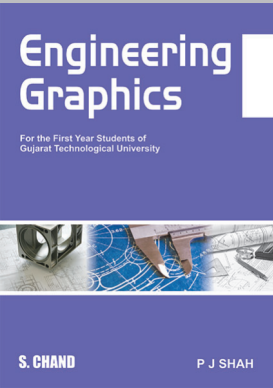
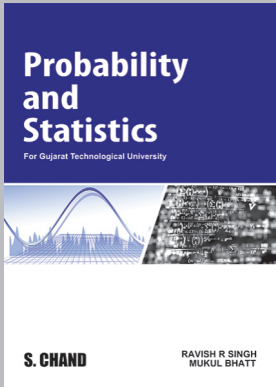
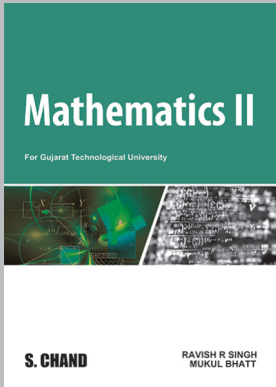
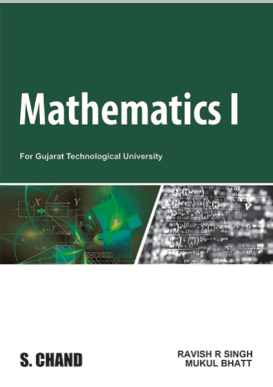
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- 1.6 Gradient
- 1.7 Divergence
- 1.8 Curl
- 1.9 Line Integrals
- 1.10 Green's Theorem in the Plane

1.1 INTRODUCTION

Vector calculus deals with the differentiation and integration of vector functions. We will learn about derivative of a vector function, gradient, divergence and curl in vector differential calculus. In vector integral calculus, we will learn about line integral, surface integral, volume integral and three theorems, namely Green's theorem, divergence theorem and Stokes' theorem. It plays an important role in the differential geometry and in the study of partial differential equations. It is useful in the study of rigid dynamics, fluid dynamics, heat transfer, electromagnetism, theory of relativity, etc.

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Complex Variables and
Partial Differential Equations

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Complex Variables
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Differential Equations

For Gujarat Technological University



Complex Variables and Partial Differential Equations

For Gujarat Technological University

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Complex Variables and Partial Differential Equations

For Gujarat Technological University

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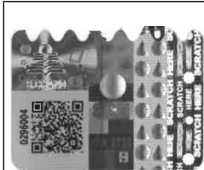
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First Edition 2023

ISBN: 978-93-5501-533-4

Product Code: H6CVD61MATH10ENAA230

PRINTED IN INDIA

By Vikas Publishing House Private Limited, Plot 20/4, Site-IV, Industrial Area Sahibabad, Ghaziabad – 201 010 and Published by S Chand And Company Limited, A-27, 2nd Floor, Mohan Co-operative Industrial Estate, New Delhi – 110 044.

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to**

Aman and Aditri

Ravish R Singh

Soumya and Siddharth

Mukul Bhatt

Preface

In mathematics, a complex variable is a variable that can take on the value of a complex number. A Partial Differential Equation (PDE) is a mathematical equation that involves two or more independent variables, an unknown function (dependent on those variables) and partial derivatives of the unknown function with respect to the independent variables. From Leonhard Euler to Joseph-Louis Lagrange, this aspect of mathematics has seen their own superstars.

“Complex Variables and Partial Differential Equations” has been designed specifically for the Gujarat Technological University (GTU) syllabus and students of engineering in their third semester.

Eight dedicated chapters are set to sequentially cover each module of the syllabus and are compounded by the ‘tutorial technique’, i.e., theory followed by example(s) so that the learner develops an increased sense of conscious intellection.

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- Completion of each section is accompanied with an exercise to test gleaning of individual subject matter.
- A rich exam-oriented pedagogy includes:
 - Close to 200 figures
 - Close to 500 in-text solved examples
 - More than 300 exercise questions

Ravish R Singh
Mukul Bhatt

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- 1.8 De Moivre's Theorem
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- 1.10 Circular and Hyperbolic Functions
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1.1 INTRODUCTION

The complex numbers are an extension of the real numbers obtained by introducing an imaginary unit i , where $i = \sqrt{-1}$. The operations of addition, subtraction, multiplication, and division are applicable on complex numbers. A negative real number can be obtained by squaring a complex number. With a complex number, it is always possible to find solutions to polynomial equations of degree more than one. Complex numbers are used in many applications, such as control theory, signal analysis, quantum mechanics, relativity, etc.

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Analysis and Synthesis

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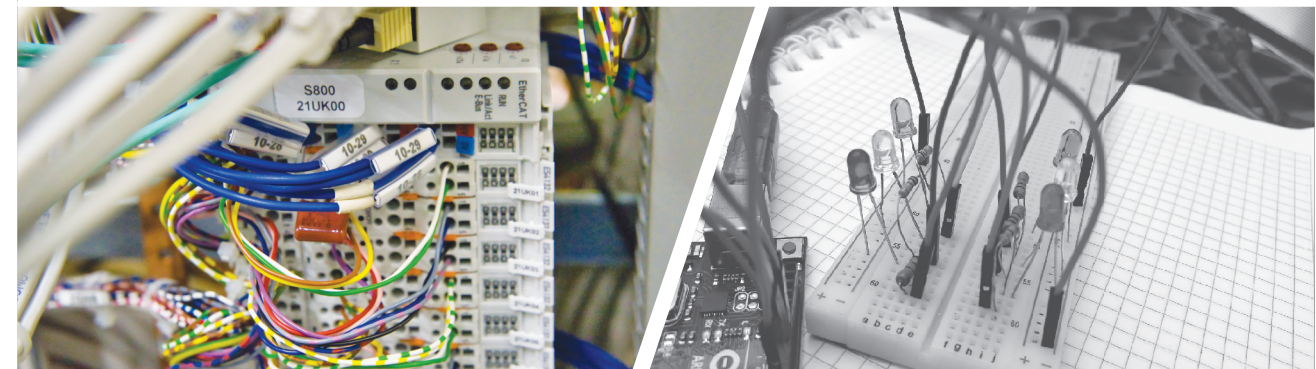
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Network Theory

Analysis and Synthesis

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NETWORK THEORY

Analysis and Synthesis

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Engineering among others for all-India curricula as well as regional curricula of universities such as Gujarat Technological University and Mumbai University among many others. Dr Singh is a member of Institute of Electrical and Electronics Engineers (IEEE), Indian Society for Technical Education (ISTE) and Institution of Electronics and Telecommunication Engineers (IETE) and has to his credit several published research papers in national and international journals. His fields of interest include Circuits, Signals and Systems and Engineering Mathematics.

NETWORK THEORY

Analysis and Synthesis

Ravish R Singh

Director

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First Edition 2023

ISBN: 978-93-5501-535-8

Product Code: H6NAS53ELEC10ENAA23O

PRINTED IN INDIA

By Vikas Publishing House Private Limited, Plot 20/4, Site-IV, Industrial Area Sahibabad, Ghaziabad – 201 010
and Published by S Chand And Company Limited, A-27, 2nd Floor, Mohan Co-operative Industrial Estate,
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My Father

Late Shri Ramsagar Singh

and

My Mother

Late Shrimati Preamsheela Singh

Preface

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ACKNOWLEDGEMENTS

My acknowledgements would be incomplete without a mention of the contribution of my family members. I am indebted to my mother and father for their lifelong inspiration. A heartfelt thanks is due to my wife, Nitu; son, Aman; and daughter, Aditri, for always motivating and supporting me in the whole project.

Any suggestions for improving the book will be gratefully acknowledged.

Ravish R Singh

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Circuit Analysis

LEARNING OBJECTIVES

After studying this chapter, you should be able to:

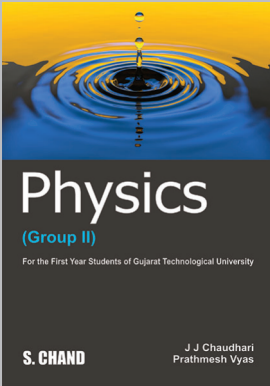
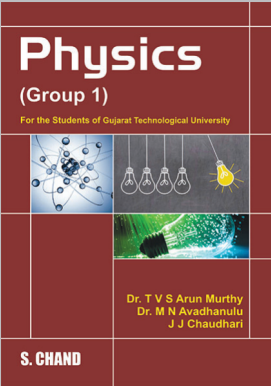
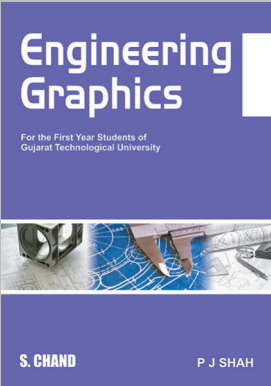
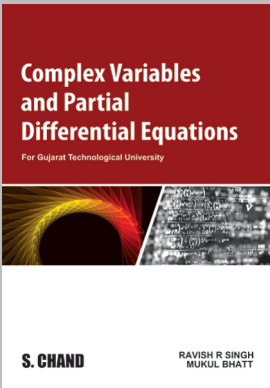
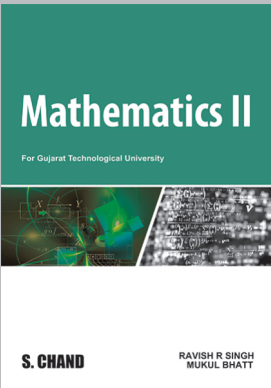
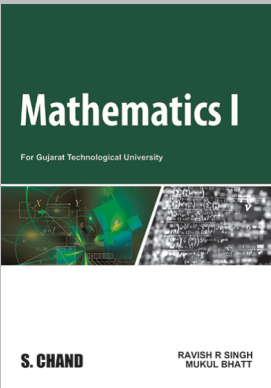
- LO 1.1** Discuss sources and their types
- LO 1.2** Define networks and circuits, linear and non-linear elements, active and passive elements, unilateral and bilateral elements, lumped and distributed elements, active and passive networks, time-invariant and time-variant networks
- LO 1.3** Use Kirchhoff's laws in solving the networks
- LO 1.4** Explain the concepts of mesh and supermesh analysis
- LO 1.5** Explain the concepts of node and supernode analysis
- LO 1.6** Analyse the networks using superposition theorem
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INTRODUCTION

In circuit analysis, we have to find currents and voltages in various parts of networks. In this chapter, we will study elementary network theorems like mesh analysis and node analysis. These methods are applicable to all types of networks. The first step in analysing networks is to apply Ohm's law and Kirchhoff's laws. The second step is the solving of these equations by mathematical tools. There are some other methods also to analyse circuits. We will also study superposition theorem, Thevenin's theorem, Norton's theorem and maximum power transfer theorem. We can find currents and voltages in various parts of the circuits with these methods.

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Probability and Statistics

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About the Authors



Ravish R Singh, B.E. (University of Mumbai), M. Tech. (IIT Bombay), Ph.D. (Faculty of Technology, University of Mumbai) is Director, Thakur Ramnarayan College of Arts and Commerce, Mumbai. A prolific author, his works range in varied subjects including, but not limited to, Engineering Mathematics, Applied Mathematics, Electrical Networks, Network Analysis and Synthesis, Electrical Engineering, Basic Electrical and Electronics

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Probability and Statistics

For Gujarat Technological University

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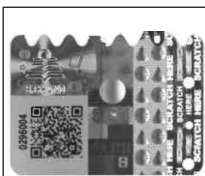
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First Edition 2023

ISBN: 978-93-5501-534-1

Product Code: H6PAS61MATH10ENAA23O

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By Vikas Publishing House Private Limited, Plot 20/4, Site-IV, Industrial Area Sahibabad, Ghaziabad – 201 010
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**Dedicated
to**

Aman and Aditri

Ravish R Singh

Soumya and Siddharth

Mukul Bhatt

Preface

Probability and Statistics are the branches of mathematics which are concerned with the laws governing random events, including the collection, analysis, interpretation and display of numerical data. From Pierre de Fermat and Blaise Pascal to John Maynard Keynes, probability has found enormous use, not really limited to mathematics anymore. Statistics in its own right has been studied by towering mathematicians such as Thomas Bayes and Pierre-Simon Laplace and our very own Prasanta Chandra Mahalanobis and CR Rao.

“Probability and Statistics” has been designed specifically for the Gujarat Technological University (GTU) syllabus and students of engineering in their third semester.

Seven dedicated chapters are set to sequentially cover each module of the syllabus and are compounded by the ‘tutorial technique’, i.e., theory followed by example(s) so that the learner develops an increased sense of conscious intellection.

This exceptional mix of theory and application caters to all types of requirements, be it the student or the teacher. Not only is the syllabus rigorously followed, but each topic has also been treated with the end-examination in sight. Concepts are well-aided with solved examples (of different complexities) so that every learner understands the topic at hand.

Salient Features:

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- Completion of each section is accompanied with an exercise to test gleaning of individual subject matter.
- A rich exam-oriented pedagogy includes:
 - More than 50 figures
 - More than 350 in-text solved examples
 - More than 325 exercise questions

**Ravish R Singh
Mukul Bhatt**

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CHAPTER 1

Probability

Chapter Outline

- 1.1 Introduction
- 1.2 Some Important Terms and Concepts
- 1.3 Definitions of Probability
- 1.4 Theorems on Probability
- 1.5 Conditional Probability
- 1.6 Multiplicative Theorem for Independent Events
- 1.7 Bayes' Theorem

1.1 INTRODUCTION

The concept of probability originated from the analysis of the games of chance. Even today, a large number of problems exist which are based on the games of chance, such as tossing of a coin, throwing of dice, and playing of cards. The utility of probability in business and economics is most emphatically revealed in the field of predictions for the future. Probability is a concept which measures the degree of uncertainty and that of certainty as a corollary.

The word *probability* or 'chance' is used commonly in day-to-day life. Daily, we come across the sentences like, 'it may rain today', 'India may win the forthcoming cricket match against Sri Lanka', 'the chances of making profits by investing in shares of Company A are very bright, etc. Each of the above sentences involves an element of uncertainty. A numerical measure of uncertainty is provided by a very important branch of mathematics called *theory of probability*. Before we study the probability theory in detail, it is appropriate to explain certain terms which are essential for the study of the theory of probability.

1.2 SOME IMPORTANT TERMS AND CONCEPTS

1. Random Experiment If an experiment is conducted, any number of times, under identical conditions, there is a set of all possible outcomes associated with it.

ENGINEERING MATHEMATICS

Volume I

For Semesters I and II

Second Edition

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McGraw Hill Education (India) Private Limited



Published by McGraw Hill Education (India) Private Limited
444/1, Sri Ekambara Naicker Industrial Estate, Alapakkam, Porur, Chennai 600 116

Engineering Mathematics (Volume I), 2e

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This edition can be exported from India only by the publishers,
McGraw Hill Education (India) Private Limited.

Print-Book Edition

ISBN (13): 978-93-90177-95-0

ISBN (10): 93-90177-95-2

E-Book Edition

ISBN (13): 978-93-90177-96-7

ISBN (10): 93-90177-96-0

1 2 3 4 5 6 7 8 9 D103074 24 23 22 21 20

Printed and bound in India.

Managing Director: *Lalit Singh*

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Typeset at APS Compugraphics, 4G, PKT 2, Mayur Vihar Phase-III, Delhi 96, and printed at

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Cover Designer: APS Compugraphics

Cover Printer:

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to our Parents**

*Late Shri Ramsagar Singh
and
Late Shrimati Preamsheela Singh*

Ravish R Singh

*Late Shri Ved Prakash Sharma
and
Late Shrimati Vidyavati Hemdan*

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- C. Inner Product Spaces

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Published by McGraw Hill Education (India) Private Limited
444/1, Sri Ekambara Naicker Industrial Estate, Alapakkam, Porur, Chennai 600 116

Engineering Mathematics (Volume II), 2e

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This edition can be exported from India only by the publishers,
McGraw Hill Education (India) Private Limited.

Print-Book Edition

ISBN (13): 978-93-90177-98-1

ISBN (10): 93-90177-98-7

E-Book Edition

ISBN (13): 978-93-90177-99-8

ISBN (10): 93-90177-99-5

1 2 3 4 5 6 7 8 9 D103074 24 23 22 21 20

Printed and bound in India.

Managing Director: *Lalit Singh*

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Typeset at APS Compugraphics, 4G, PKT 2, Mayur Vihar Phase-III, Delhi 96, and printed at

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Cover Designer: APS Compugraphics

Cover Printer:

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CIN: U80302TN2010PTC111532

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to our Parents**

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Ravish R Singh

*Late Shri Ved Prakash Sharma
and
Late Shrimati Vidyavati Hemdan*

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Complex Variables and Partial Differential Equations

Second Edition

Gujarat Technological University 2020

About the Authors



Ravish R Singh is presently Director at Thakur Ramnarayan College of Arts and Commerce, Mumbai. He obtained a BE degree from University of Mumbai in 1991, an MTech degree from IIT Bombay in 2001, and a PhD degree from Faculty of Technology, University of Mumbai, in 2013. He has published several books with McGraw Hill Education (India) on varied subjects like *Engineering Mathematics*, *Applied Mathematics*, *Electrical Networks*, *Network Analysis and Synthesis*, *Electrical Engineering*, *Basic Electrical and Electronics Engineering*, etc., for all-India curricula as well as regional curricula of some universities like Gujarat Technological University, Mumbai University, Pune University, Jawaharlal Nehru Technological University, Anna University, Uttarakhand Technical University, and Dr A P J Abdul Kalam Technical University. Dr Singh is a member of IEEE, ISTE, and IETE, and has published research papers in national and international journals. His fields of interest include Circuits, Signals and Systems, and Engineering Mathematics.



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Complex Variables and Partial Differential Equations, 2e (GTU–2020)

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Print-Book Edition

ISBN (13): 978-93-90219-89-6

ISBN (10): 93-90219-89-2

E-Book Edition

ISBN (13): 978-93-90219-90-2

ISBN (10): 93-90219-90-6

1 2 3 4 5 6 7 8 9 D103074 24 23 22 21 20

Printed and bound in India.

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Typeset at APS Compugraphics, 4G, PKT 2, Mayur Vihar Phase-III, Delhi 96, and printed at
Rajkamal Electric Press, Plot No. 2, Phase-IV, Kundli, Haryana.

Cover Designer: APS Compugraphics

Cover Printer: Rajkamal Electric Press

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to**

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Ravish R Singh

Soumya and Siddharth

Mukul Bhatt

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NETWORK THEORY

Analysis and Synthesis

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Ravish R Singh is presently Director at Thakur Ramnarayan College of Arts and Commerce, Mumbai. He obtained a BE degree from University of Mumbai in 1991, an MTech degree from IIT Bombay in 2001, and a PhD degree from Faculty of Technology, University of Mumbai, in 2013. He has published several books with McGraw Hill Education (India) on varied subjects like *Engineering Mathematics*, *Applied Mathematics*, *Electrical Networks*, *Network Analysis and Synthesis*, *Electrical Engineering*, *Basic Electrical and Electronics Engineering*, etc., for all-India curricula as well as regional curricula of some universities like Gujarat Technological University, Mumbai University, Pune University, Jawaharlal Nehru Technological University, Anna University, Uttarakhand Technical University, and Dr A P J Abdul Kalam Technical University. Dr Singh is a member of IEEE, ISTE, and IETE, and has published research papers in national and international journals. His fields of interest include Circuits, Signals and Systems, and Engineering Mathematics.

NETWORK THEORY

Analysis and Synthesis

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McGraw Hill Education (India) Private Limited



Published by McGraw Hill Education (India) Private Limited
444/1, Sri Ekambara Naicker Industrial Estate, Alapakkam, Porur, Chennai 600 116

Network Theory: Analysis and Synthesis (MU 2020)

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Print-Book Edition

ISBN (13): 978-81-947400-0-1

ISBN (10): 81-947400-0-2

E-Book Edition

ISBN (13): 978-81-947400-1-8

ISBN (10): 81-947400-1-0

1 2 3 4 5 6 7 8 9 D103074 24 23 22 21 20

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Typeset at APS Compugraphics, 4G, PKT 2, Mayur Vihar Phase-III, Delhi 96, and printed at

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My Father

Late Shri Ramsagar Singh

and

My Mother

Late Shrimati Preamsheela Singh

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Probability and Statistics

Second Edition

Gujarat Technological University 2020

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Probability and Statistics

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444/1, Sri Ekambara Naicker Industrial Estate, Alapakkam, Porur, Chennai 600 116

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ISBN (13): 978-93-90219-91-9

ISBN (10): 93-90219-91-4

E-Book Edition

ISBN (13): 978-93-90219-92-6

ISBN (10): 93-90219-92-2

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Typeset at APS Compugraphics, 4G, PKT 2, Mayur Vihar Phase-III, Delhi 96, and printed at
Rajkamal Electric Press, Plot No. 2, Phase-IV, Kundli, Haryana.

Cover Designer: APS Compugraphics

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Second Edition

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Second Edition

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Network Analysis and Synthesis, 2e

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This edition can be exported from India only by the publishers,
McGraw Hill Education (India) Private Limited.

Print-Book Edition

ISBN (13): 978-93-5316-672-4

ISBN (10): 93-5316-672-1

E-Book Edition

ISBN (13): 978-93-5316-673-1

ISBN (10): 93-5316-673-X

1 2 3 4 5 6 7 8 9 D103074 23 22 21 20 19

Printed and bound in India.

Managing Director: *Lalit Singh*

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Typeset at APS Compugraphics, 4G, PKT 2, Mayur Vihar Phase-III, Delhi 96, and printed at

Cover Image Source: Design Pics / Corey Hochachka

Cover Designer: APS Compugraphics

Cover Printer:

Visit us at: www.mheducation.co.in

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CIN: U80302TN2010PTC111532

Toll Free Number: 1800 103 5875

Dedicated to

My Father

Late Shri Ramsagar Singh

and

My Mother

Late Shrimati Preamsheela Singh

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Complex Variables and Partial Differential Equations

Gujarat Technological University 2019

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Complex Variables and Partial Differential Equations

Gujarat Technological University 2019

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McGraw Hill Education (India) Private Limited



Published by McGraw Hill Education (India) Private Limited
444/1, Sri Ekambara Naicker Industrial Estate, Alapakkam, Porur, Chennai 600 116

Complex Variables and Partial Differential Equations, GTU–2019

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McGraw Hill Education (India) Private Limited.

ISBN (13): 978-93-5316-725-7

ISBN (10): 93-5316-725-6

1 2 3 4 5 6 7 8 9 D103074 23 22 21 20 19

Printed and bound in India.

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Typeset at APS Compugraphics, 4G, PKT 2, Mayur Vihar Phase-III, Delhi 96, and printed at

Cover Designer: APS Compugraphics

Cover Printer:

Visit us at: www.mheducation.co.in

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CIN: U80302TN2010PTC111532

Toll Free Number: 1800 103 5875

**Dedicated
to**

Aman and Aditri

Ravish R Singh

Soumya and Siddharth

Mukul Bhatt

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Probability and Statistics

Gujarat Technological University 2019

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McGraw Hill Education (India) Private Limited



Published by McGraw Hill Education (India) Private Limited
444/1, Sri Ekambara Naicker Industrial Estate, Alapakkam, Porur, Chennai 600 116

Probability and Statistics, GTU–2019

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ISBN (13): 978-93-5316-755-4

ISBN (10): 93-5316-755-8

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Typeset at APS Compugraphics, 4G, PKT 2, Mayur Vihar Phase-III, Delhi 96, and printed at

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**Dedicated
to**

Aman and Aditri

Ravish R Singh

Soumya and Siddharth

Mukul Bhatt

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Basic Electrical Engineering

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Basic Electrical Engineering

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to
My son, *Aman*
and
daughter, *Aditri*

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ISBN (10): 93-89691-82-6

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Typeset at APS Compugraphics, 4G, PKT 2, Mayur Vihar Phase-III, Delhi 96, and printed at

Cover Designer: APS Compugraphics

Cover Printer:

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**Dedicated
to**

Aman and Aditri

Ravish R Singh

Soumya and Siddharth

Mukul Bhatt

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444/1, Sri Ekambara Naicker Industrial Estate, Alapakkam, Porur, Chennai 600 116

Basic Electrical Engineering, 3e

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McGraw Hill Education (India) Private Limited.

1 2 3 4 5 6 7 8 9 D103074 22 21 20 19 18

Printed and bound in India.

ISBN (13): 978-93-5316-172-9

ISBN (10): 93-5316-172-X

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Typeset at APS Compugraphics, 4G, PKT 2, Mayur Vihar Phase-III, Delhi 96, and printed at

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to
My son, *Aman*
and
daughter, *Aditri*

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Circuit Theory and Networks

Analysis and Synthesis

SECOND EDITION

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444/1, Sri Ekambara Naicker Industrial Estate, Alapakkam, Porur, Chennai 600 116

Circuit Theory and Networks—Analysis and Synthesis, 2e (MU 2018)

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Printed and bound in India.

ISBN (13): 978-93-5316-173-6

ISBN (10): 93-5316-173-8

Director—Science & Engineering Portfolio: *Vibha Mahajan*
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Typeset at APS Compugraphics, 4G, PKT 2, Mayur Vihar Phase-III, Delhi 96, and printed at

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My Father

Late Shri Ramsagar Singh

and

My Mother

Late Shrimati Preamsheela Singh

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Mathematics-1

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1 2 3 4 5 6 7 8 9 D103074 22 21 20 19 18

Printed and bound in India.

Print-Book

ISBN (13): 978-93-5316-280-1

ISBN (10): 93-5316-280-7

E-Book

ISBN (13): 978-93-5316-281-8

ISBN (10): 93-5316-281-5

Director—Science & Engineering Portfolio: *Vibha Mahajan*

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Typeset at APS Compugraphics, 4G, PKT 2, Mayur Vihar Phase-III, Delhi 96, and printed at

Cover Designer: APS Compugraphics

Cover Image Source: Shutterstock

Cover Printer:

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**Dedicated
to**

Aman and Aditri

Ravish R Singh

Soumya and Siddharth

Mukul Bhatt

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Mathematics-2

Gujarat Technological University 2018

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Gujarat Technological University 2018

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Published by McGraw Hill Education (India) Private Limited
444/1, Sri Ekambara Naicker Industrial Estate, Alapakkam, Porur, Chennai 600 116

Mathematics-2, GTU-2018

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1 2 3 4 5 6 7 8 9 D103074 22 21 20 19 18

Printed and bound in India.

ISBN (13): 978-93-5316-489-8

ISBN (10): 93-5316-489-3

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Typeset at APS Compugraphics, 4G, PKT 2, Mayur Vihar Phase-III, Delhi 96, and printed at

Cover Designer: APS Compugraphics

Cover Image Source: Shutterstock

Cover Printer:

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**Advanced
Engineering Mathematics
Fourth Edition**

Gujarat Technological University 2018

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Advanced Engineering Mathematics

Fourrth Edition

Gujarat Technological University 2018

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Published by McGraw Hill Education (India) Private Limited
444/1, Sri Ekambara Naicker Industrial Estate, Alapakkam, Porur, Chennai 600 116

Advanced Engineering Mathematics, 4e, GTU–2018

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Printed and bound in India.

Print-Book

ISBN (13): 978-93-5316-103-3

ISBN (10): 93-5316-103-7

E-Book

ISBN (13): 978-93-5316-104-0

ISBN (10): 93-5316-104-5

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Typeset at APS Compugraphics, 4G, PKT 2, Mayur Vihar Phase-III, Delhi 96, and printed at

Cover Designer: APS Compugraphics

Cover Printer:

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Preface

Mathematics is a key area of study in any engineering course. A sound knowledge of this subject will help engineering students develop analytical skills, and thus enable them to solve numerical problems encountered in real life, as well as apply mathematical principles to physical problems, particularly in the field of engineering.

Users

This book is designed for the 2nd year GTU engineering students pursuing the course Advanced Engineering Mathematics, SUBJECT CODE: 2130002 in their 3rd Semester. It covers the complete GTU syllabus for the course on Advanced Engineering Mathematics, which is common to all the engineering branches.

Objective

The crisp and complete explanation of topics will help students easily understand the basic concepts. The tutorial approach (i.e., teach by example) followed in the text will enable students develop a logical perspective to solving problems.

Features

Each topic has been explained from the examination point of view, wherein the theory is presented in an easy-to-understand student-friendly style. Full coverage of concepts is supported by numerous solved examples with varied complexity levels, which is aligned to the latest GTU syllabus. Fundamental and sequential explanation of topics are well aided by examples and exercises. **The solutions of examples are set following a ‘tutorial’ approach, which will make it easy for students from any background to easily grasp the concepts.** Exercises with answers immediately follow the solved examples enforcing a practice-based approach. We hope that the students will gain logical understanding from solved problems and then reiterate it through solving similar exercise problems themselves. The unique blend of theory and application caters to the requirements of both the students and the faculty. Solutions of GTU examination questions are incorporated within the text appropriately.

Highlights

- Crisp content strictly as per the latest GTU syllabus of Advanced Engineering Mathematics (Regulation 2014)
- Comprehensive coverage with lucid presentation style
- Each section concludes with an exercise to test understanding of topics
- Solutions of GTU examination questions from 2012 to 2018 present appropriately within the chapters and on companion web link
- Rich exam-oriented pedagogy:
 - Solved examples within chapters: 475
 - Solved GTU questions within chapters: 247
 - Unsolved exercises: 571
 - MCQs at the end of chapters: 121
 - MCQs on web link: 50

Chapter Organization

The content spans the following six chapters which wholly and sequentially cover each module of the syllabus.

- ❑ **Chapter 1** introduces Some Special Functions.
- ❑ **Chapter 2** discusses Fourier Series and Fourier Integral.
- ❑ **Chapter 3** presents Ordinary Differential Equations and Applications.
- ❑ **Chapter 4** covers Series Solution of Differential Equations.
- ❑ **Chapter 5** deals with Laplace Transforms and Applications.
- ❑ **Chapter 6** presents Partial Differential Equations and Applications.

Acknowledgements

We are grateful to the following reviewers who reviewed various chapters of the script or previous editions of the book and generously shared their valuable comments:

Mukesh Shimpi	<i>BVM Engineering College, Anand, Gujarat</i>
Manokamna Agrawal	<i>Silver Oak College of Engineering and Technology, Ahmedabad, Gujarat</i>
Som Sahni	<i>ITM Universe Vadodara</i>
Vijay Solanki	<i>GEC Bharuch</i>
Foram Rajdev	<i>Marwadi Education Foundation Group of Institutions, Rajkot</i>
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Usha Bag	<i>Shree L R Tiwari College of Engineering, Thane, Maharashtra</i>

We would also like to thank all the staff at McGraw Hill Education (India), especially Vibha Mahajan, Shalini Jha, Hemant K Jha, Tushar Mishra, Satinder Singh Baveja, Taranpreet Kaur and Anuj Shrivastava for coordinating with us during the editorial, copyediting, and production stages of this book.

Our acknowledgements would be incomplete without a mention of the contribution of all our family members. We extend a heartfelt thanks to them for always motivating and supporting us throughout the project.

Constructive suggestions for the improvement of the book will always be welcome.

Ravish R Singh
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ROADMAP TO THE SYLLABUS

This text is useful for

SUBJECT CODE: 2130002 – Advanced Engineering Mathematics

Module 1: Introduction to Some Special Functions

Gamma function; Beta function; Bessel function; Error function and complementary error function; Heaviside's function; Pulse unit height and duration function; Sinusoidal pulse function; Rectangle function; Gate function; Dirac's Delta function; Signum function; Sawtooth wave function; Triangular wave function; Half-wave rectified sinusoidal function; Full rectified sine wave; Square wave function.

GO TO

CHAPTER 1: Introduction to Some Special Functions

Module 2: Fourier Series and Fourier Integral

Periodic function; Trigonometric series; Fourier series; Functions of any period; Even and odd functions; Half-range expansion; Forced oscillations; Fourier integral.

GO TO

CHAPTER 2: Fourier Series and Fourier Integral

Module 3: Ordinary Differential Equations and Applications

First order differential equations: basic concepts; Geometric meaning of $y' = f(x, y)$ Direction fields; Exact differential equations; Integrating factor; Linear differential equations; Bernoulli equations; Modeling: Orthogonal trajectories of curves; Linear differential equations of second and higher order: Homogeneous linear differential equations of second order; Modeling: Free oscillations; Euler-Cauchy Equations; Wronskian; Nonhomogeneous equations; Solution by undetermined coefficients; Solution by variation of parameters; Modeling: Free Oscillations, Resonance and electric circuits; Higher order linear differential equations; Higher order homogeneous equations with constant coefficient; Higher order nonhomogeneous equations. Solution by $[1/f(D)] f(x)$ method for finding particular integral.

GO TO

CHAPTER 3: Ordinary Differential Equations and Applications

Module 4: Series Solution of Differential Equations

Power series method; Theory of power series methods; Frobenius method.

**GO TO****CHAPTER 4: Series Solution of Differential Equations****Module 5: Laplace Transforms and Applications**

Definition of the Laplace transform; Inverse Laplace transform; Linearity; Shifting theorem; Transforms of derivatives and integrals; Differential equations; Unit step function; Second shifting theorem; Dirac's delta function; Differentiation and integration of transforms; Convolution and integral equations; Partial fraction differential equations; Systems of differential equations.

**GO TO****CHAPTER 5: Laplace Transforms and Applications****Module 6: Partial Differential Equations and Applications**

Formation of PDEs; Solution of partial differential equations $f(x, y, z, p, q) = 0$; Nonlinear PDEs of first order; Some standard forms of nonlinear PDEs; Linear PDEs with constant coefficients; Equations reducible to homogeneous linear form; Classification of second-order linear PDEs; Separation of variables; Use of Fourier series; D'Alembert's solution of the wave equation; Heat equation: Solution by Fourier series and Fourier integral.

**GO TO****CHAPTER 6: Partial Differential Equations and Applications**

Mathematics IV

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444/1, Sri Ekambara Naicker Industrial Estate, Alapakkam, Porur, Chennai 600 116

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McGraw Hill Education (India) Private Limited.

1 2 3 4 5 6 7 8 9 D103074 22 21 20 19 18

Printed and bound in India.

ISBN (13): 978-93-87886-30-8

ISBN (10): 93-87886-30-1

Director—Science & Engineering Portfolio: *Vibha Mahajan*

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Typeset at APS Compugraphics, 4G, PKT 2, Mayur Vihar Phase-III, Delhi 96, and printed at

Cover Printer:

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To Our Parents**

*Late Shri Ramsagar Singh
and
Late Shrimati Premsheela Singh*

Ravish R Singh

*Late Shri Ved Prakash Sharma
and
Late Shrimati Vidyavati Hemdan*

Mukul Bhatt

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Printed and bound in India.

ISBN (13): 978-93-5316-505-5

ISBN (10): 93-5316-505-9

Managing Director: *Lalit Singh*

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Typeset at APS Compugraphics, 4G, PKT 2, Mayur Vihar Phase-III, Delhi 96, and printed at

Cover Designer: APS Compugraphics

Cover Printer:

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Soumya and Siddharth

Mukul Bhatt

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1 2 3 4 5 6 7 8 9 D103074 22 21 20 19 18

Printed and bound in India.

Print-Book Edition

ISBN (13): 978-93-5316-266-5

ISBN (10): 93-5316-266-1

E-Book Edition

ISBN (13): 978-93-5316-267-2

ISBN (10): 93-5316-267-X

Managing Director: *Lalit Singh*

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Typeset at APS Compugraphics, 4G, PKT 2, Mayur Vihar Phase-III, Delhi 96, and printed at

Cover Designer: APS Compugraphics

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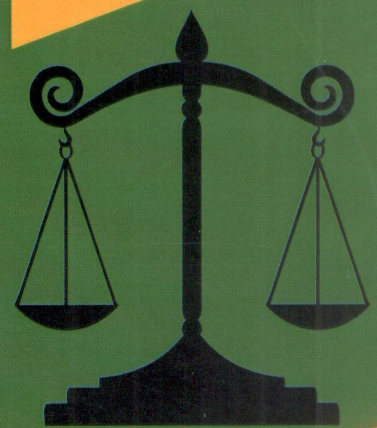
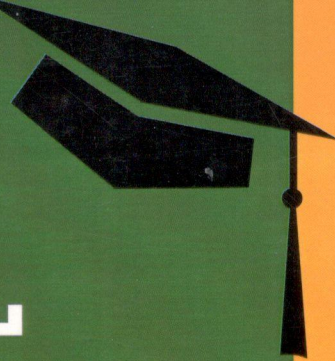
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LEGAL DYNAMICS

A Compilation of Recent
Developing Trends in Law
and Society

Edited by
Dr. A.K. SINGH & RAJWANT RAO



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This edition published in 2018
by

INFINITY PUBLISHING

4574/15, Ansari Road, Opp. Happy School
Daryaganj, New Delhi - 110002 (India)
Phone No: +91-11- 40115111, 23264395, 9899982737
Email: sales@infinitypublishing.co.in
www.infinitypublishing.co.in

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Title: LEGAL AUDIT
ISBN: 978-93-87910-46-1
Author: Various
Subject: Law

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Printed and Bound In India

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2. IMPLEMENTATION OF ISO 9001:2015 INTERNATIONAL STANDARD IN TEACHING LEARNING PROCESS OF TRCL

Ravish R Singh¹ & Mukul Bhatt²

INTRODUCTION

The adoption of a quality management system is a strategic decision for an organization that can help to improve its overall performance and provide a sound basis for sustainable development initiatives.

The potential benefits to an organization of implementing a quality management system based on ISO 9001:2015 International Standard are:

- a) the ability to consistently provide products and services that meet customer and applicable statutory and regulatory requirements;
- b) facilitating opportunities to enhance customer satisfaction;
- c) addressing risks and opportunities associated with its context and objectives;
- d) the ability to demonstrate conformity to specified quality management system requirements. This International

ISO 9001:2015 International Standard employs the process approach, which incorporates the Plan-Do-Check-Act (PDCA) cycle and risk-based thinking. The process approach enables an organization to plan its processes and their interactions. The PDCA cycle enables an organization to ensure that its processes are adequately resourced and managed, and that opportunities for improvement are determined and acted on. Risk-based thinking enables an organization to determine the factors that could cause its processes and its quality management system to deviate from the planned results, to put in place preventive controls to minimize negative effects and to make maximum use of opportunities as they arise. Consistently meeting requirements and addressing future needs and expectations poses a challenge for organizations in an increasingly dynamic and complex environment. To achieve this objective, the organization might find it necessary to adopt various forms of improvement in addition to correction and continual improvement, such as breakthrough change, innovation and re-organization.

The organization shall establish, implement, maintain and continually improve a quality management system, including the processes needed and their interactions, in accordance with the requirements of ISO 9001:2015 International Standard.

The organization shall determine the processes needed for the quality management system and their application throughout the organization, and shall:

- a) determine the inputs required and the outputs expected from these processes;
- b) determine the sequence and interaction of these processes;
- c) determine and apply the criteria and methods (including monitoring, measurements and related performance indicators) needed to ensure the effective operation and control of these processes;
- d) determine the resources needed for these processes and ensure their availability;
- e) assign the responsibilities and authorities for these processes;
- f) address the risks and opportunities as determined in accordance with the requirements;
- g) evaluate these processes and implement any changes needed to ensure that these processes achieve their intended results;
- h) improve the processes and the quality management system.

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CASE STUDY OF TRCL

The Teaching Learning Process in TRCL is based on effective delivery of Lectures, tutorial, Practical Trainings and DPC as per timetable, academic calendar and teaching plan. In case of any shortcoming in teaching learning process, necessary remedial work is carried out in coordination with the Principal. Learning of the student is monitored by the faculty and the Principal on the basis of continuous evaluation. All lectures, tutorials, practical trainings and DPC are conducted as per the teaching plan and are reviewed time-to-time to improve the efficiency and effectiveness of the teaching-learning process during semester. Students' learning is validated through mock activities, continuous evaluation (CE) and semester end examinations (SEE). The process is monitored at two levels viz. Faculty and Principal to avoid lapses. Mentoring System is developed for solving academic and stress related issues of students. The faculty are encouraged to use ICT for effective teaching with Learning Management Systems (LMS), E-learning resources etc.

a) Inputs required and the outputs expected from Teaching-Learning Process

Input required	From	Output expected	To
Number of students Number of divisions Faculty Timetable Teaching plan	Admission	Daily attendance record Defaulter list Syllabus coverage	Examination Library Co-curricular and Extra-curricular Activities Placement and Higher Education

b) Sequence and interaction of Teaching Learning Process

1. Academic calendar TRCL/IP/02/FRM/01 is prepared by the Principal at least seven days before the commencement of the semester by considering the list of holidays.
2. Meetings of the faculty are conducted by the Principal to allocate the workload. Class, divisions, as well as courses, are allocated before the start of the semester in accordance with guidelines issued by UGC and MU regarding faculty workload.
3. The allocation of workload TRCL/IP/02/FRM/02 is communicated to the Timetable Committee by the Principal.
4. Master timetable TRCL/IP/02/FRM/03, Division-wise timetable TRCL/IP/02/FRM/04 and Faculty-wise timetable TRCL/IP/02/FRM/05 are prepared by the Timetable Committee at least three days before the commencement of the semester
5. Subject specific term work module/assessment modes like extension/field or experimental work, short quiz, objective test, open book etc. and written assignments, case study, judgment analysis, projects, papers and exhibits etc. as are designed by the faculty.
6. Semester-wise teaching plan is prepared by the faculty
7. A Faculty Diary (TRCL/IP/02/REG/01) is maintained by each faculty.
8. Classes are engaged in accordance with the Division wise Time Table TRCL/IP/02/FRM/04.
9. The attendance of students is maintained in Attendance Record TRCL/IP/02/FRM/10 on regular basis.

10. Lecture record is maintained in Work Report TRCL/IP/02/FRM/09 on a daily basis by individual faculty.
11. Work Report TRCL/IP/02/FRM/09 is checked on weekly basis by the Principal.
12. In case of any faculty being absent, a substitute faculty is sent to the class by the Principal and a record is maintained by the faculty who was absent in the Leave Load Arrangement form TRCL/IP/02/FRM/13.
13. Academic loss in the subject due to the leave of the faculty is compensated by arranging extra lecture. A record is maintained in Leave Load Arrangement form TRCL/IP/02/FRM/13.
14. Extra lecture is arranged by the faculty to complete the syllabus if required and a record is maintained in Extra Lecture Load/Remedial Work Report TRCL/IP/02/FRM/14.
15. Remedial lectures are arranged for academically weaker students (identified on the basis of their result and performance in class) and records are maintained in Extra Lecture Load/Remedial Work Report TRCL/IP/02/FRM/14.
16. Tutorials/Practical Trainings/DPC sessions are conducted by the faculty as scheduled in the Teaching Plan.
17. Tutorials/Practical Trainings/DPC records are maintained in Work Report TRCL/IP/02/FRM/09 on daily basis by individual faculty.
18. Tutorials/Practical Trainings/DPC attendance of students is maintained in Attendance Record TRCL/IP/02/FRM/10.

c) The criteria and methods needed to ensure the effective operation and control of Teaching Learning Process

1. Monthly Teaching-Learning Process Review is done by the Principal. Academic conduct review consists of a report of class attendance, syllabus coverage, students' performance, students' defaulter list etc.
2. Review report is submitted to the Principal for approval and necessary action is taken for key findings.
3. Attendance is monitored by the Principal during the midterm/semester review and the remedial action is taken so that all the students comply the attendance criteria.
4. Internal Assessment Examination is conducted as per Academic Calendar and results are analyzed.

d) The resources needed for Teaching Learning

Process and to ensure their availability

1. Identification of resources required is initiated by the Principal for smooth conduction of the Teaching-Learning process.
2. Requirements for infrastructural facilities, computer (hardware and software) and human resources are identified by the Principal well in advance on yearly basis.
3. Requirements for Teaching and Learning resources such as reference books, textbooks, e-books, etc. are given to Librarian for the procurements before the start of the semester as per the Library Process TRCL/IP/04.
4. All other resources (viz. stationary, displays, teaching aids, furniture and fixture etc.) related to TLP are arranged before the start of the semester.
5. Time Table Committee is appointed at the beginning of the academic year by the Principal and all the members are informed accordingly.

e) Assigning the responsibilities and authorities for Teaching Learning Process

Activity	Responsibility & Authority
Identification of resources and its management	Principal
Preparation of timetable	Timetable Committee
Preparation of teaching plan	Faculty
Conducting lectures, tutorials, practical trainings and DPC	Faculty
Overall monitoring and control	Principal

Risk	Mitigation	Opportunity
Lack of co-ordination between Principal/Faculty/ Students	The final timetable is distributed to the faculty and students. The division-wise timetable is displayed on the college website.	The smooth conduct of all the lectures by the faculty as mentioned in the timetable Parents awareness about the college timetable
Non-availability of faculty due to leave	Lecture adjustment is done by the faculty approved by the Principal.	The subject can be taken by another faculty interested in that particular subject. Increased flexibility and adaptability of the faculty helpful for the institution.
Resignation of faculty during semester	At least 10-15 bio-data should be ready in the administrative office. A copy must be submitted to the timetable committee for rectification.	Well qualified and experienced faculty can be recruited from the available bio-data. Training to faculty can increase their work potential and diligence.
No improvement after remedial lectures	Weekly evaluation of students	Academic improvement of the students

f) Risks and Opportunities in Teaching Learning Process

g) Evaluation of Teaching Learning Process and implementing any changes needed to ensure that the process achieve intended results

1. Cumulative monthly attendance analysis TRCL/IP/02/FRM/16 is submitted by the attendance

committee on fifth of every month during the semester to the Principal for display on the notice boards. In case of the defaulter, corrective actions are initiated by the class in charges as per the guidelines given by the Principal from time-to-time.

2. Cumulative monthly syllabus coverage report TRCL/IP/02/FRM/17 is submitted by Faculty in-charge on fifth of every month during the semester to the Principal. Corrective actions are initiated and completed by the Principal as needed.

3. Corrective actions for detected NCO are initiated and completed by Principal as explained in Control of Nonconforming Output Process TRCL/MP/09.

h) Improvement in the Teaching Learning Process and the quality management system

1. Strict adherence to the Academic Calendar for the various activities of Teaching-Learning Process
2. Enhancing learning and development through holistic teaching approach
3. Creating a competitive environment for the students' overall interaction
4. Scope for precise teaching plan gained through experience
5. Self-introspection by faculty for innovation and creativity in Teaching-Learning Process
6. Benchmarking of innovative and best practices

CONCLUSION

In this paper a case study of Teaching Learning Process of TRCL is correlated with the procedures specified in ISO 9001: 2015.